Introduction of the Liénard-Wiechert correction to the particle simulation of relativistic plasmas

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We discuss the implementation of time retardation in the relativistic simulation of a plasma. We show that this has to be done at two different levels: first, with the introduction of retarded charge and current densities in the potential integration and, second, with the implementation of the Liénard-Wiechert correction, in the form of velocity dependent denominators. While the first step is often included in the particle calculation of plasmas, the second is, generally, neglected. The introduction of the Liénard-Wiechert correction in the philosophy of plasma particle-in-cell codes is, however, nontrivial. In this paper, we propose an extension of these codes that includes this relativistic correction, and we show explicitly how it can be adapted to one-dimensional calculations. Finally we present results of the extended code corresponding to a plasma interacting with a very intense electromagnetic wave. As a consequence of the field anisotropy induced by the Liénard-Wiechert correction, we find a dramatic enhancement in the high-order frequencies of the electromagnetic field scattered by the plasma target. [S1063-651X(98)09509-9]

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I. INTRODUCTION

In the past two decades we have witnessed an extraordinary development in laser techniques, which have lead to a rapid growth in the field of nonlinear optics beyond the perturbative regime. Several new aspects have been investigated, both theoretically and experimentally [1]. The fundamental mechanisms involved in the field harmonic generation and the ionization of single-electron atoms seem now understood with the aid of massive computer calculations, from which some simple analytical models can be derived. The attention is now focused on more complex systems, like many electron atoms, molecules, or clusters, for which the exact numerical integration of the equations of motion becomes a formidable task. Among these complex systems, solids are being intensively studied at present [2]. There are a number of reasons for this interest: first, it is well known that the intensity of the scattered radiation is, generally, a growing function with the number of scatterers, therefore the reflected field from a solid target is orders of magnitude more intense than that of a single atom. Second, solids are much easier than gases to handle in the laboratory. The experimental convenience is, however, balanced by the complex theoretical treatment of such systems. While the exact integration has been attempted only for very specific cases [3], approximated models give good insight provided the laser intensity is high enough to ionize the target, and therefore, to create a plasma [4,5].

The numerical integration of a plasma target impinged by a laser beam can be performed either from the point of view of the plasma as a charged fluid, or by considering it as a set of interacting charged particles. The simplicity of the latter approach is, probably, one of the reasons why it is so successful among the scientific community. The so-called particle-in-cell (PIC) codes discretize the charge density into an ensemble of quasiparticles with a chosen spatial extension and the corresponding charge. The particles' dynamics is governed by the Lorentz equation, and the electromagnetic field is integrated from the charge and current densities calculated from the particle distribution [6,7].

II. RELATIVISTIC FIELD CORRECTION IN THE PARTICLE-IN-CELL CALCULATIONS

The standard PIC codes incorporate relativity in the integration of the Lorentz equation for the particles. Although this is a major correction, it is only partial since the relativistic approach should be included also when integrating Maxwell equations. In principle, one must expect to find a retarded solution for the fields, which can only be obtained by an integration in the four-dimensional space-time, and not by an integration in the three-dimensional space. The retarded solution for the transverse field has been used previously [6], but is generally neglected for the longitudinal field.

A natural way of including retardation is to integrate Maxwell equations for the potentials in the Lorentz gauge,

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi=-4\pi\rho, \tag{1}$$

$$\nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A}=-\frac{4\pi}{c}\vec{j}. \tag{2}$$

These equations have an integral solution of the form [10]

$$\phi(\vec{r},t)=\int\frac{\rho(\vec{r}\,',t')}{|\vec{r}-\vec{r}\,'|}\bigg|_{t'}d\vec{r}\,',\quad \vec{A}(\vec{r},t)=\frac{1}{c}\int\frac{\vec{j}(\vec{r}\,',t')}{|\vec{r}-\vec{r}\,'|}\bigg|_{t'}d\vec{r}\,'. \tag{3}$$

where $t'=t-|\vec{r}-\vec{r}\,'|/c$. From the particle viewpoint, the charge density associated with an ensemble of charges may be written as

$$\begin{aligned}\rho(\vec{r},t)&=\int d\vec{r}\,'\sum_m q_m\delta[\vec{r}\,'-\vec{r}_m(t)]\\&=\int d\vec{r}\,'\sum_m q_m\delta\left[\vec{r}\,'-\vec{r}_m(0)-\int_0^t\vec{v}_m(\tau)d\tau\right],\end{aligned} \tag{4}$$

where we have replaced the charges' coordinates by their time-integral representation. The common approach in plasma particle calculations is to compute the charge density from Eq. (4) discretized in a spatial grid,

$$\rho(\vec{r}_i,t)=\frac{1}{\Delta V}\sum_{\substack{m\\ x_m^\alpha=x_i^\alpha}} q_m, \tag{5}$$

ΔV being the volume element of the spatial grid, and where i labels the spatial cell in the grid. The summatory of Eq. (5) is restricted to those particles whose position $\vec{r}_m(t')$ at $t'=t-|\vec{r}-\vec{r}_m(t')|/c$ is within the cell located at $\vec{r}_i$, i.e., $x_m^\alpha(t')=x_i^\alpha$ in 4-vector notation.

In a particle-in-cell code, $\rho(\vec{r},t')$ is computed at all previous times using Eq. (5). Therefore, one might want to calculate the potentials from the grid version of Eq. (3). However, this is not a correct procedure. For instance, for a single particle, we would obtain a retarded Coulomb potential for the scalar potential,

$$\phi(\vec{r}_i,t)=\frac{q}{|\vec{r}_i-\vec{r}\,'(t')|}\quad(\text{wrong}). \tag{6}$$

In contrast, the correct formula for the relativistic one-particle potential is given by the well-known *Liénard-Wiechert* expression,

$$\phi(\vec{r}_i,t)=\left[1-\frac{\vec{v}\,'(t')\cdot\vec{n}}{c}\right]^{-1}\frac{q}{|\vec{r}_i-\vec{r}\,'(t')|} \tag{7}$$

where $\vec{n}=[\vec{r}_i-\vec{r}\,'(t')]/|\vec{r}_i-\vec{r}\,'(t')|$, the unit vector pointing to the direction of observation. The reason for the misleading of Eq. (6) is that we have implicitly taken as unity the spatial integral of the retarded delta function [8]

$$\int\delta[\vec{r}\,'-\vec{r}_m(t')]d\vec{r}\,'=\int\frac{1}{1-(\vec{v}_m\cdot n)/c}\bigg|_{t'(\vec{u})}\delta(\vec{u})d\vec{u}\neq 1. \tag{8}$$

Therefore, the relativistic corrections in the solution of Maxwell equations appear at two different levels. First, with the introduction of retardation, as in Eq. (3) and, second, in taking into account the Liénard-Wiechert velocity denominators when computing the charge and current densities from an ensemble of particles. A direct way to introduce both levels is to compute the potential associated with every charge and calculating the total potential as the superposition of these single-particle potentials,

$$\begin{aligned}\phi(\vec{r},t)&=\sum_m \phi_m(\vec{r},t)\\&=\sum_m\frac{1}{1-\vec{v}_m(t_m')\cdot\vec{n}_m(t_m')/c}\,\frac{q_m}{|\vec{r}-\vec{r}_m(t_m')|}.\end{aligned} \tag{9}$$

Although, in principle, this is a correct way of calculating the whole potential, it lacks one of the major advantages of the PIC algorithms: By calculating the charge and current densities associated by a set of charges, and using them to compute the associated fields, they reduce drastically the time requirements of the code. This is clear by noting that Eq. (9) requires a scan over the whole set of particles to compute the potential at a single point in the space $\vec{r}$. PIC calculations, on the other hand, compute the charge and current densities associated with the set of charges, which requires a single scan over the set of particles, and then integrate Maxwell equations in space, which requires a single scan over the spatial grid. It is, therefore, justified to make the effort to introduce the relativistic corrections discussed in the preceding section into the PIC calculation philosophy. We will next show how this can be done, provided we introduce some approximations.

First, let us factorize the summation of Eq. (9) into a set of partial summations of particles that share the same volume element at the same time, $(\vec{r}_m,t_m)=(\vec{r}\,',t')$ or $x_m^\alpha=x'^\alpha$ in 4-vector notation. In the following, we will assume the ions are fixed, which is a reasonable approximation for the short laser pulses considered in this paper (it must be pointed out, however, that the extension of the following treatment to include ions or other plasma species is trivial). Since the ionic potential is constant in time, the only dynamic contribution comes from the plasma electrons. The charge of the particles being the same, we have

$$\phi_e(\vec{r},t)=\int d\vec{r}\,'\frac{\rho_e(\vec{r}\,',t')}{|\vec{r}-\vec{r}_m|}\frac{1}{N(\vec{r}\,',t')}\sum_{\substack{m\\ x_m^\alpha=x'^\alpha}}\frac{1}{1-\vec{v}_m(t')\cdot\vec{n}/c}, \tag{10}$$

where $N(\vec{r}\,',t')$ is the number of particles sharing the same volume element at time t', and the subindex e stands for electrons. To obtain the total potential, the ionic part should be added. Defining the averaged velocity of the electrons at each volume element as

$$\vec{v}(\vec{r}\,',t')=\frac{1}{N(\vec{r}\,',t')}\sum_{\substack{m\\ x_m^\alpha=x'^\alpha}}\vec{v}_m(t') \tag{11}$$

we have $\vec{v}_m(t')=\vec{v}(\vec{r}\,',t')+\Delta\vec{v}_m(t')$. Substituting in Eq. (10), the term in the denominator may be expressed as a Taylor series in the velocity fluctuations, $\Delta\vec{v}_m(t')/c$,

$$\begin{aligned}\frac{1}{1-\vec{v}_m(t')\cdot\vec{n}/c}\simeq\frac{1}{1-\vec{v}(t')\cdot\vec{n}/c}\Bigg\{1+\frac{\Delta\vec{v}_m(t')\cdot\vec{n}/c}{1-\vec{v}(t')\cdot\vec{n}/c}\\+\left[\frac{\Delta\vec{v}_m(t')\cdot\vec{n}/c}{1-\vec{v}(t')\cdot\vec{n}/c}\right]^2+\cdots\Bigg\}.\end{aligned} \tag{12}$$

Performing the summation of Eq. (10), the first term of the Taylor series amounts to $N(\vec{r}\,',t')$, while the second vanishes because of the definition of $\Delta\vec{v}_m(t')$. The third term in the Taylor series contains the following sum,

$$\sum_{\substack{m\\ x_m^{\alpha}=x'^{\alpha}}} [\Delta\vec{v}_m(t')\cdot\vec{n}]^2 = \frac{1}{2}\sum_{\substack{m\\ x_m^{\alpha}=x'^{\alpha}}} \Delta v_m^2(t') = \frac{3N(\vec{r}',t')}{2m} kT(\vec{r}',t'), \tag{13}$$

where we have assumed an isotropic distribution of the velocity fluctuations, and we have introduced the definition of a local time-dependent nonrelativistic temperature $T(\vec{r}',t')$ [9]. Keeping the series expansion to second order, the potential approximation suitable for particle-in-cell calculations is

$$\phi_e(\vec{r},t) \simeq \int \frac{1}{|\vec{r}-\vec{r}'|}\frac{\rho_e(\vec{r}',t')}{1-\vec{v}(t')\cdot\vec{n}/c} \times\left\{1+\frac{3kT(\vec{r}',t')/2mc^2}{[1-\vec{v}(t')\cdot\vec{n}/c]^2}\right\} d\vec{r}'. \tag{14}$$

In principle, this truncated expression is valid only if $|\Delta\vec{v}(t')|<|c-v(t')|$. This is ensured for particles faster than the average by the relativistic dynamic itself. The velocity distribution in the relativistic regime is, however, strongly asymmetric and particles with velocities well below the mean value may exist. Although these particles do not fulfill the truncation condition, nevertheless their contribution to integral (14) is not essential, since their velocity-dependent denominator is large.

The expression for the potential vector $\vec{A}$ can be obtained following the same steps: first, we calculate the total potential as the sum of the single-particle Liénard-Wiechert expressions and, second, we introduce the mean velocity

$$\vec{A}(\vec{r},t) = \int d\vec{r}'\frac{\rho_e(\vec{r}',t')}{c|\vec{r}-\vec{r}_m|}\frac{1}{N(\vec{r}',t')} \times \sum_{\substack{m\\ x_m^{\alpha}=x'^{\alpha}}} \frac{\vec{v}(t')+\Delta\vec{v}_m(t')}{1-[\vec{v}(t')+\Delta\vec{v}_m(t')]\cdot\vec{n}/c}. \tag{15}$$

Since ions are supposed to be fixed, the only contribution to the total vector potential comes from the electrons. If we define the current density as

$$\vec{j}(\vec{r},t) = \sum_{\substack{m\\ x_m^{\alpha}=x'^{\alpha}}} q_m\vec{v}_m(t')\delta(\vec{r}-\vec{r}') = \sum_{\substack{m\\ x_m^{\alpha}=x'^{\alpha}}} q_m\delta(\vec{r}-\vec{r}')[\vec{v}(t')+\Delta\vec{v}_m(t')] = \vec{v}(\vec{r},t)\rho_e(\vec{r},t) \tag{16}$$

then the first term in the sum of Eq. (15) leads to a mean-velocity contribution to the vector potential

$$\vec{A}_v(\vec{r},t) \simeq \int \frac{1}{c|\vec{r}-\vec{r}'|}\frac{\vec{j}(\vec{r}',t')}{1-\vec{v}(t')\cdot\vec{n}/c} \times\left\{1+\frac{3kT(\vec{r}',t')/2mc^2}{[1-\vec{v}(t')\cdot\vec{n}(t')/c]^2}\right\} d\vec{r}'. \tag{17}$$

The second term of the summatory in Eq. (15) can be evaluated as

$$\sum_{\substack{m\\ x_m^{\alpha}=x'^{\alpha}}} \frac{\Delta\vec{v}_m(t')}{1-[\vec{v}(t')+\Delta\vec{v}_m(t')]\cdot\vec{n}/c} \simeq \sum_{m x_m^{\alpha}=x'^{\alpha}} \frac{\Delta\vec{v}_m(t')(\Delta\vec{v}_m(t')\cdot\vec{n})}{[1-\vec{v}(t')\cdot\vec{n}/c]^2}. \tag{18}$$

For an isotropic distribution of the velocity fluctuations, we might find

$$\vec{A}_{\Delta v}(\vec{r},t) \simeq \int \frac{1}{|\vec{r}-\vec{r}'|}\frac{\rho_e(\vec{r}',t')\vec{n}}{[1-\vec{v}(t')\cdot\vec{n}(t')/c]^2}\frac{3}{2}\frac{kT(\vec{r}',t')}{mc^2} d\vec{r}'. \tag{19}$$

The complete vector potential being $\vec{A}(\vec{r},t)=\vec{A}_v(\vec{r},t)+\vec{A}_{\Delta v}(\vec{r},t)$.

Equations (14), (17), and (19) give solutions for the field potentials in the approximation of a low temperature plasma. Their integration introduces a new complexity in PIC codes and precludes the direct use of the spatial Fourier transform or the spatial finite difference methods since now space and time are coupled variables. This affects especially the cases in which the symmetry of the problem reduces the spatial dimensionality. For instance, we will show in the next section that in the case of 1D problems the potential integrals involve actually two dimensions. However, as will be seen, some approximations may be used for certain geometries to reduce again the dimensionality to one.

III. FORM OF THE POTENTIALS FOR ONE-DIMENSIONAL CALCULATIONS

Particle-in-cell simulations in one dimension have been proven to give good insight into a number of the fundamental processes involved in the interaction of plasmas with strong fields. Basically, they integrate the equations in a one

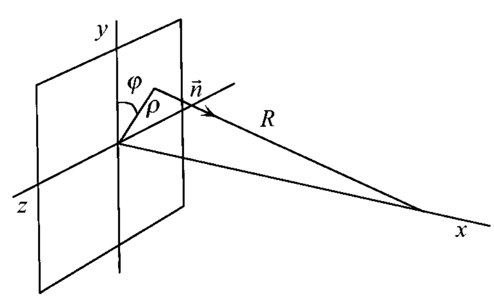

FIG. 1. Geometry considered to find the 1D form of the electromagnetic potentials. The charge and current densities are assumed constant on each yz plane.

dimensional space, while keeping the three dimensional nature of the vectors involved, i.e., the electromagnetic fields and the particles' velocities. Although the transversal dynamics is neglected, and therefore effects like self-focusing and filamentation are missing, the drastic reduction in computation time allows the integration of the equations of motion over a space extension of several wavelengths for interaction times of several optical periods, which is extremely difficult for multidimensional calculations with dense grids. We shall find now the expressions for the retarded electromagnetic potentials suitable for one-dimensional PIC.

The physical interpretation of a one-dimensional problem is to consider that all the functions involved have a constant value along the plane perpendicular to the chosen spatial coordinate. One dimensional particles, therefore, correspond to uniform charged planes in the three dimensional space, whose electrostatic attraction does not have the Coulombic $1/R^2$ form, but the constant force between the plates of a capacitor [7]. For this to be realistic, one dimensional PIC codes are restricted to the case of loose focused laser beams with rather uniform cross section.

Assuming a cold plasma, i.e., $T(\vec{r},t)=0$, the potential of Eq. (14) can be expressed in the following form:

$$\phi_e(x,t)\simeq\int dx'\int\int dy'dz'\frac{1}{|\vec{r}-\vec{r}\,'|}\times\frac{\rho_e^2(x',t')}{\rho_e(x',t')-\vec{j}(x',t')\cdot\vec{n}/c}, \tag{20}$$

where we assume the functions to be constant in the yz plane, and where we have substituted the mean velocity $\vec{v}(t')$ in terms of the current and charge densities, following Eq. (16). For convenience, we define the distance to the observation point as $R=|\vec{r}-\vec{r}\,'|$ and the cylindrical coordinates $(\varrho,\varphi,x-x')$; see Fig. 1. Introducing these definitions in Eq. (20), and using $\vec{n}=(\vec{r}-\vec{r}\,')/R$, we have

$$\phi_e(x,t)\simeq\int dx'\int\int \varrho\, d\varrho\, d\varphi\frac{\rho_e^2(x',t')}{R}\left[\rho_e(x',t')-j_x(x',t')\frac{x-x'}{Rc}-j_y(x',t')\frac{\varrho\cos\varphi}{Rc}-j_z(x',t')\frac{\varrho\sin\varphi}{Rc}\right]^{-1}. \tag{21}$$

Noting that $R=\sqrt{(x-x')^2+\varrho^2}$ and that $R=c(t-t')$, then $RdR=\varrho\, d\varrho$ if we keep x' constant and

$$\phi_e(x,t)\simeq c^2\int dx'\int_{-\infty}^{t-|x-x'|/c}dt'\int_0^{2\pi}d\varphi\,\rho_e^2(x',t')\left\{c\rho_e(x',t')-j_x(x',t')\frac{x-x'}{c(t-t')}-[j_y(x',t')\cos\varphi+j_z(x',t')\sin\varphi]\sqrt{1-\left[\frac{x-x'}{c(t-t')}\right]^2}\right\}^{-1}. \tag{22}$$

The integral over the angle φ can be easily calculated, giving the final expression for the scalar potential as

$$\phi_e(x,t)\simeq-2\pi c^2\int dx'\int_{-\infty}^{t-|x-x'|/c}dt'\,\rho_e^2(x',t')\left\{\left[c\rho_e(x',t')-j_x(x',t')\frac{x-x'}{c(t-t')}\right]^2-[j_y^2(x',t')+j_z^2(x',t')]\left[1-\left(\frac{x-x'}{c(t-t')}\right)^2\right]\right\}^{-1/2}. \tag{23}$$

In a similar way one can obtain the expression for the vector potential as

$$\vec{A}(x,t)\simeq-2\pi c\int dx'\int_{-\infty}^{t-|x-x'|/c}dt'\,\rho_e(x',t')\vec{j}(x',t')\left\{\left[c\rho_e(x',t')-j_x(x',t')\frac{x-x'}{c(t-t')}\right]^2-[j_y^2(x',t')+j_z^2(x',t')]\left[1-\left(\frac{x-x'}{c(t-t')}\right)^2\right]\right\}^{-1/2}. \tag{24}$$

The general expression for each component Λ of the 4-vector potential is

$$\Lambda(x,t)\simeq\int dx'\mathcal{F}_\Lambda(x',x,t-|x-x'|/c), \tag{25}$$

where $\mathcal{F}_\Lambda(x',x,t-|x-x'|/c)$ is the time integral of Eqs. (23) and (24). It is clear that, despite our one-dimensional approximation, the integration of these potentials has a 2D complexity, since we have two free variables, x and x'. The two dimensionality of the problem reflects the anisotropy of the electric field induced by the relativistic velocity. There are, however, situations that allow for a one dimensional reduction of Eqs. (23) and (24). Consider, for instance, the case of a thin plasma slab whose width is much smaller than the focal section of the laser spot. In this situation $R\gg|x$

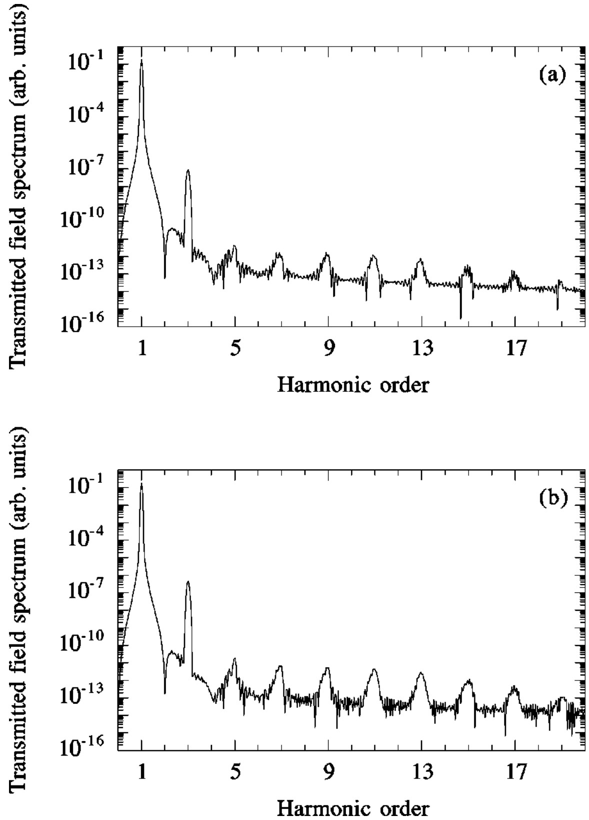

FIG. 2. Harmonic spectra of the field transmitted by a plasma slab. The plasma density is 1.65 times the critical density. The laser intensity is $\simeq 4\times 10^{16}$ W/cm^2. (a) Harmonic spectrum calculated from a PIC code without the Liénard-Wiechert correction, (b) calculated with the LW correction.

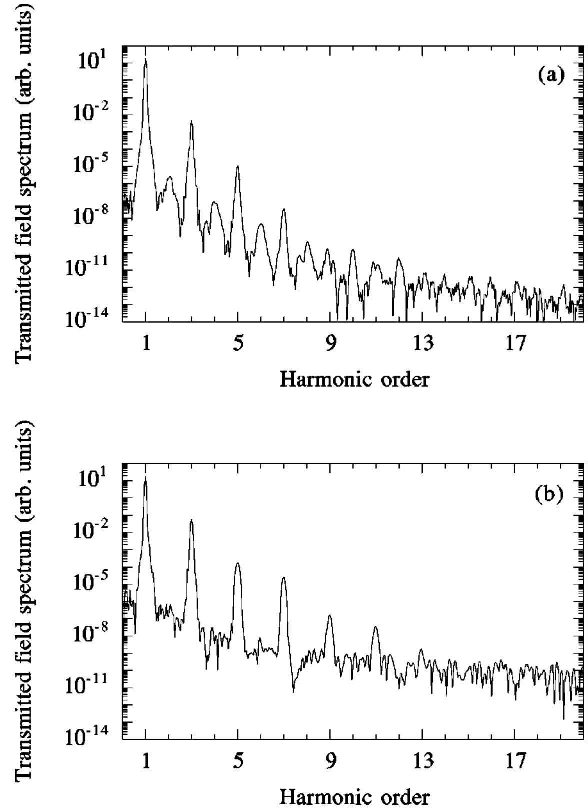

FIG. 3. The same as Fig. 2, but with the laser intensity increased to $\simeq 4\times 10^{18}$ W/cm^2.

$-x'|$ for most of the region where the integral $\mathcal{F}_\Lambda(x',x,t-|x-x'|/c)$ extends. We can, therefore approximate Eq. (23) as

$$\phi_e(x,t) \simeq \int dx' \mathcal{F}_\phi\left(x',t-\frac{|x-x'|}{c}\right)$$
$$\simeq -2\pi \int dx' \int_{-\infty}^{t-|x-x'|/c}$$
$$\times \frac{c^2\rho_e^2(x',t')dt'}{\{c^2\rho_e^2(x',t')-[j_y^2(x',t')+j_z^2(x',t')]\}^{1/2}} \tag{26}$$

and $\vec{A}(x,t)$ can also be approximated accordingly. Also, for the case in which the incident electromagnetic wave is aimed perpendicularly to the plasma surface, the electric field will accelerate the charges mainly in the transversal coordinate (y or z). In this case, it is expected that $|j_y^2(x',t')+j_z^2(x',t')| > |j_x^2(x',t')|$, thus ensuring the correctness of the approximation. The integration of the retarded expressions of Eq. (26), and the correspondent to $\vec{A}(x,t)$, can be performed in one dimension, by discretizing the spatial axis into cells of length $\Delta x = c\Delta t$, and splitting into two contributions on the left and on the right side of x [6].

IV. NUMERICAL RESULTS

We have introduced retardation effects into our 1D PIC code with the approximations discussed in the previous section. Our program, therefore, calculates the electromagnetic fields by integrating the retarded potentials from the Maxwell equations, taking into account the Liénard-Wiechert (LW) correction in the form of Eq. (26). On the other hand, the particles' dynamics, which is governed by the Lorentz equation, is also integrated relativistically. Although three dimensional velocities are considered, we only take into account one dimension in space, in the philosophy of the so-called 1D3V PIC codes (one dimensional in space and three dimensional in velocity) [6,7].

As discussed above, the 1D approximation describes a plasma slab of infinite extension in the transverse direction, interacting with an electromagnetic wave, also uniform in the transverse direction. We will consider, therefore, the longitudinal axis as the only spatial dimension of our calculation. The electromagnetic field is assumed to be incident perpendicularly to the plasma surface, propagating along the spatial dimension considered. In order to preserve consistency with the approximations discussed in the previous section, our plasma slab must be thin in terms of the laser wavelength λ, therefore we will consider a target thickness of 0.1λ, with $\lambda \simeq 0.9$ μm (the laser frequency being $\omega = 0.05$ a.u.).

Figure 2 shows the harmonic spectra of the field transmitted by a plasma slab whose density is 1.65 times the critical density. The laser intensity is $\simeq 4\times 10^{16}$ W/cm^2 (field amplitude of 1 a.u.). The harmonic spectrum calculated from a PIC code without the LW correction for the potentials is shown in Fig. 2(a), while the spectrum calculated including

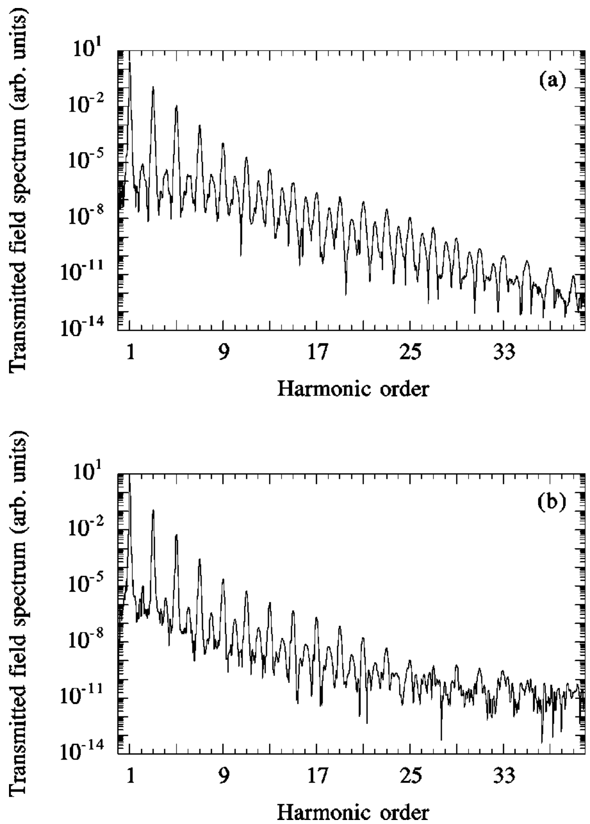

FIG. 4. Reflected field spectra for the cases of Fig. 3.

LW correction is plotted in Fig. 2(b). As is apparent, the relativistic correction is almost insignificant at this laser intensity. Note that the ratio between the nonrelativistic free electron quiver velocity to the light speed is $a_0=0.14$, and therefore we are in the weak relativistic regime. It is only worth mentioning a slight increase of the harmonic intensities for the LW corrected case (b). The same slight increase has been observed in the spectrum of the reflected field, not shown in this paper, when the LW correction is included.

The situation changes dramatically when we increase the intensity in two orders of magnitude. Figure 3 shows the harmonic spectra of the transmitted field for the same parameters as in Fig. 2, but with the intensity increased to 4×10^{18} W/cm^2. Now the nonrelativistic quiver velocity would be higher than the light speed, which means that we are in a strongly relativistic regime. The comparison with the calculation including LW correction [Fig. 3(b)], and without it [Fig. 3(a)], shows a clear increase of the intensity of the harmonics radiated in the corrected case. The increase seems to be more pronounced for the higher harmonics, which can be orders of magnitude. This high-order harmonic intensity enhancement is also apparent in the reflected field, which is shown in Fig. 4.

The reason underlying the increase of the high-order harmonic intensities when we include the LW correction may be understood by inspection of Fig. 5. This figure shows a density plot of the longitudinal electric field E_x as a function of time (vertical axis) and along the integration space length (horizontal axis). As previously, Figs. 5(a) and 5(b) are calculated without and including LW, respectively. One can notice the increase of the longitudinal electric field oscillations inside the plasma slab when the LW correction is taken into account. The electric field oscillations depicted in both pictures correspond to the capacitorlike field induced by the surface charge originated by the longitudinal quiver of the negative charges, as a result of the $\vec{v}\times\vec{B}$ term in the Lorentz force. It is the basic mechanism underlying the so-called moving mirror models [4,5].

The increase in the amplitude of the oscillations in the LW corrected calculation must be, however, attributed to a different mechanism. To analyze it, let us come back to the relativistic field emitted by a charge. The introduction of the LW velocity dependent denominator breaks the isotropy of the Coulomb potential. If one considers the electric field associated with LW potential, it can be split into two contributions [10], one proportional to the charge's velocity and the other to the acceleration. The near field is dominated by the velocity component, which can be written for one particle in terms of its *instantaneous* position $\vec{r}\,'(t)$ as

$$\vec{E}(\vec{r},t)=\frac{1}{\gamma^2[1-(v/c)^2\sin^2\psi]^{3/2}}\frac{q\vec{n}}{|\vec{r}-\vec{r}\,'|^2}, \tag{27}$$

where the terms on the right-hand side of the equation should

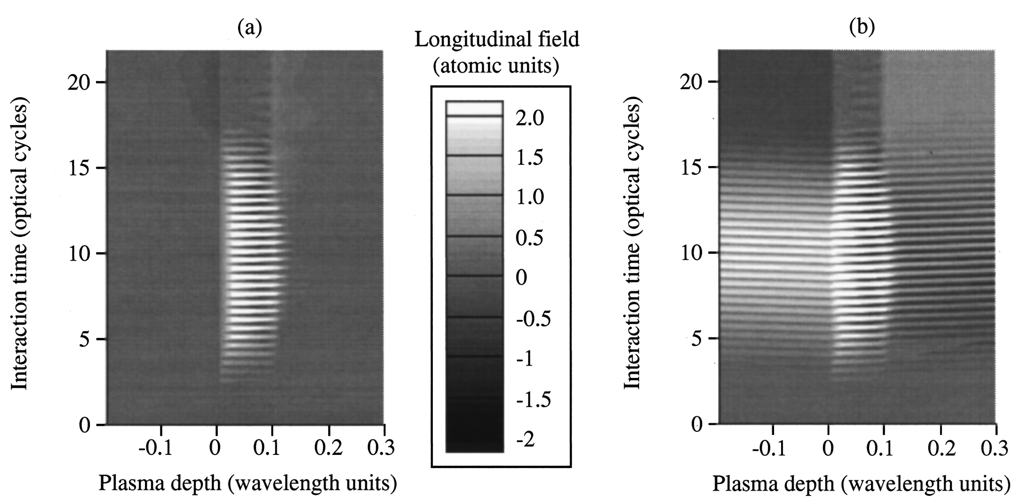

FIG. 5. Time evolution of the longitudinal electric field for the case of Fig. 3. (a) shows the result neglecting the Liénard-Wiechert correction, and (b) including it.

be understood to be computed at the present time t, and where $\psi = \cos^{-1}(\vec{n}\cdot\vec{v})$. This field is strongly anisotropic, and can be understood in terms of a Coulombic field with an anisotropic effective charge. The effective charge observed in the direction orthogonal to the particle's displacement is much larger than the charge at rest. In our calculations, particles quiver mostly in the direction parallel to the field polarization, i.e., the y coordinate. As they quiver, their effective charge observed in the x direction fluctuates from the rest charge to higher values. When the effective charge is greater than the rest charge, the ion background is not able to compensate the longitudinal field. As a result, plasma oscillations are enhanced and reflected as new high-frequency components of the scattered radiation. It should also be noted from Fig. 4(b) that as the effective charge increases the plasma neutrality is lost, and a residual longitudinal field may be detected at some distance of the target. This residual field, however, has the r^{-2} near field dependence, and it is damped at a short distance from the plasma surface.

V. CONCLUSION

We have analyzed the introduction of the retarded fields in the particle simulation of plasmas. As we have discussed, this implies two different steps. First the use of retarded density functions in the integral of the potentials, and second the introduction of the Liénard-Wiechert (LW) correction. We think that this latter fact has been usually underestimated in the particle calculations of plasmas. As a result of the anisotropic denominator included in the LW correction, we show that an enhancement of the longitudinal field oscillations inside the plasma is expected. In turn, this will give rise to plasma charge oscillations that result in a growth in the intensity of the high-frequency components of the scattered field.

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